

The effective potential of composite diquark fields and the spectrum of resonances in dense QCD

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The effective potential of composite diquark fields responsible for color symmetry breaking in cold very dense QCD, in which long-range interactions dominate, is derived. The spectrum of excitations and the universality class of this dynamics are described.

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Recently, there has been considerable interest in the study of the color superconducting phase of cold dense QCD [1–10] (for recent reviews, see Ref. [11]). The color superconducting quark matter may exist in the interior of neutron stars, with baryon number densities exceeding a few times the normal nuclear density $n_0 \simeq 0.17 \text{ fm}^{-3}$. Also, such matter could be created in accelerators by heavy ion collisions.

The Ginzburg–Landau (GL) effective action method has been extremely successful in studying ordinary superconductivity of metals [12]. Recently, a similar approach has been utilized in the study of color superconductivity [3,4]. However, there the effective action was postulated based on symmetry and renormalization group arguments, and not derived from the microscopic theory, QCD.

Following the original approach of Gorkov [12], it would be of a great interest to derive the effective action in color superconductivity directly from QCD. In this letter, we make a step in realizing this program and derive the effective potential for the order parameter of color superconductivity in cold dense QCD at such high baryon densities when the fermion pairing in the diquark channel dominates over that in the chiral one [1,2] and when long-range interactions dominate [4,5]. For this purpose, we will utilize the method of Ref. [13], which was originally used for the derivation of the effective action in quenched strong-coupling QED₄ (see also Ref. [14]) and then was successfully applied to QED₃ [15], quenched QED₄ in a magnetic field [16], and to some other models [17].

The crucial feature in the dynamics of cold dense QCD, pointed recently in Refs. [4,5] (see also Ref. [6]), is the presence of the long-range interactions mediated by the unscreened gluon modes of the magnetic type. This point essentially distinguishes the dynamics of color superconductivity from that in the BCS theory of superconductivity in metals. In particular, this makes the derivation of the effective action in color superconductivity more complicated than the derivation of the GL effective action from the BCS theory.

Our derivation of the effective potential in dense QCD will be based on the recent analysis of color superconductivity in the framework of the Schwinger–Dyson (SD) equations [7–9]. In this way, we will describe the universality class of the dynamics in cold dense QCD and, in particular, get insight into the character of the spectrum of excitations.

As we shall see below, the universality class of the system at hand is that connected with long-range non-isotropic forces, producing a bifermion condensate. We will see that this class resembles (although does not quite coincide with) that of quenched QED₄ in a constant magnetic field [18,19]. The scaling law for the order parameter X in these two models has the following form:

$$X = \Lambda_{eff} f(z), \quad f(z) \sim \exp\left(-\frac{C}{\sqrt{z}}\right), \quad (1)$$

and C is some constant. Here z is a generic notation for parameters of a theory, such as a coupling constant, temperature, the number of fermion flavors, *etc.*. In QED₄ in a magnetic field B , the effective cutoff Λ_{eff} is proportional to $|eB|^{1/2}$ and z is the QED running coupling at the scale $|eB|^{1/2}$. In cold dense QCD, which is of main interest here, Λ_{eff} is proportional to the chemical potential μ and z is the running QCD coupling constant α_s at the scale μ .

The critical value z_c is zero both in cold dense QCD and in QED₄ in a magnetic field. This is because in these two models, strong interactions are provided by the effective dimensional reduction $3 + 1 \rightarrow 1 + 1$ in the dynamics of fermion pairing [12,18].

One should expect that the long-range interaction in dense QCD leads to the existence of an infinite number of resonances in different channels. In particular, as we will see, there is indeed an infinite number of resonances in the

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channel with the quantum numbers of the Nambu–Goldstone (NG) bosons. It will be shown that this in turn leads to a rather unconventional form of the effective potential: it is a multibranched function of the bifermion condensate and has an infinite number of local minima. It reduces to the conventional Coleman–Weinberg potential [20] only in the vicinity of the global minimum.

So, let us consider dense QCD with two light flavors in the chiral limit. The Lagrangian density reads

$$\mathcal{L}_{QCD} = \bar{\psi} (i\gamma^\mu D_\mu + \mu\gamma^0) \psi - \frac{1}{2} \text{Tr} (F_{\mu\nu} F^{\mu\nu}) + \mathcal{L}_{gf} + \mathcal{L}_{FP}, \quad (2)$$

where \mathcal{L}_{gf} and \mathcal{L}_{FP} are the gauge fixing and the Faddeev–Popov ghost terms. The covariant derivative is defined in a usual way, $D_\mu = \partial_\mu - ig_s A_\mu^A T^A$, and μ is the chemical potential.

Below, it will be convenient to work with the eight component Majorana spinors, $\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi \\ \psi^C \end{pmatrix}$ where $\psi^C = C\bar{\psi}^T$ and C is a charge conjugation matrix, defined by $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ and $C = -C^T$. The fermion part of the Lagrangian density (2) could be rewritten as follows:

$$\mathcal{L}_{fer} = \bar{\Psi} \begin{pmatrix} i\gamma^\mu D_\mu + \mu\gamma^0 & 0 \\ 0 & i\gamma^\mu \tilde{D}_\mu - \mu\gamma^0 \end{pmatrix} \Psi, \quad \tilde{D}_\mu = \partial_\mu + ig_s A_\mu^A (T^A)^T. \quad (3)$$

In order to derive the one-particle irreducible (1PI) effective action for the local composite field $\hat{\phi}_c(x) = 1/2 \varepsilon^{ij} \varepsilon_{abc} \bar{\psi}_a^i(x) \gamma^5 C \left(\bar{\psi}_b^j(x) \right)^T$ (here a, b, c and i, j are the color and flavor indices, respectively), whose vacuum expectation value defines the order parameter in the theory, we need to consider the corresponding generating functional,

$$iW(J_c) = \ln \int d\Psi d\bar{\Psi} dA_\mu \exp \left[i \int d^4x \left(\mathcal{L}_{QCD} + \frac{1}{2} J_c \varepsilon^{ij} \varepsilon_{abc} \bar{\psi}_a^i \gamma^5 C \left(\bar{\psi}_b^j \right)^T + c.c. \right) \right]. \quad (4)$$

When the functional $W(J_c)$ is known, the calculation of the effective action (potential) of interest reduces to performing the Legendre transform with respect to the external source J_c ,

$$\Gamma(\phi_c) = W(J_c) - \int d^4x [J_c(x) \phi_c(x) + c.c.], \quad (5)$$

where $\phi_c(x) = \langle 0 | \hat{\phi}_c(x) | 0 \rangle_J$, and the subscript J implies that $\phi_c(x)$ is related to the theory with a source J_c . It is assumed that the source J_c in Eq. (5) is the function of the field ϕ_c , obtained by inverting the expression,

$$\frac{\delta W}{\delta J_c(x)} = \phi_c(x). \quad (6)$$

For the purposes of calculating the effective potential of the field ϕ_c , it is sufficient to restrict ourselves to the case of a constant (in space–time) external source, $J_c(x) = \text{Const}$. In addition, using the freedom of global color transformations, it is always possible to fix the orientation of the source in the color space along the third direction, *i.e.*, $J_1 = J_2 = 0$ and $J_3 \equiv j \neq 0$. Finally, the baryon symmetry allows us to choose j to be real.

In the theory with the external source, the inverse of the bare fermion propagator reads

$$G_0^{-1} = -i \begin{pmatrix} \hat{p} + \mu\gamma^0 & J \\ \gamma^0 J^\dagger \gamma^0 & \hat{p} - \mu\gamma^0 \end{pmatrix}, \quad J_{ab}^{ij} = j \varepsilon^{ij} \varepsilon_{ab3} \gamma^5. \quad (7)$$

Upon neglecting the wave function renormalizations [5–9], the inverse of the full fermion propagator, G^{-1} , would be the same as that in Eq. (7) but with J_{ab}^{ij} replaced by $\Sigma_{ab}^{ij}(p) = \Delta(p) \varepsilon^{ij} \varepsilon_{ab3} \gamma^5$. By inverting it, we obtain the following expression for the fermion propagator:

$$G = i \begin{pmatrix} R_1(p)^{-1} & -(\hat{p} + \mu\gamma^0)^{-1} \Sigma R_2(p)^{-1} \\ -(\hat{p} - \mu\gamma^0)^{-1} \gamma^0 \Sigma^\dagger \gamma^0 R_1(p)^{-1} & R_2(p)^{-1} \end{pmatrix}, \quad (8)$$

where

$$R_1(p) = (\hat{p} + \mu\gamma^0) - \Sigma (\hat{p} - \mu\gamma^0)^{-1} \gamma^0 \Sigma^\dagger \gamma^0, \quad (9)$$

$$R_2(p) = (\hat{p} - \mu\gamma^0) - \gamma^0 \Sigma^\dagger \gamma^0 (\hat{p} + \mu\gamma^0)^{-1} \Sigma. \quad (10)$$

As is clear from the definition of the fermion propagator, $\Delta(p)$ is directly related to the value of the gap in the fermion spectrum in the color superconducting phase. At the same time, it is also related to the vacuum expectation value of the diquark field. Indeed, by making use of its definition, we obtain

$$\phi \equiv \phi_3 = \varepsilon^{ij} \varepsilon_{ab3} \text{tr} \left[(G_{12})_{ab}^{ij} \gamma^5 \right] \simeq -8i \int \frac{d^4 p}{(2\pi)^4} \frac{\Delta(p)}{p_0^2 - (|\vec{p}| - \mu)^2 - \Delta^2}. \quad (11)$$

(Note that this expression, up to the change of notations, $\Delta \rightarrow \phi_{l-}^+ = -\phi_{r+}^+$, would remain the same if the gap ansatz of Refs. [21,9] is used. In notation of Ref. [8], $\Delta \rightarrow \Delta_1$.) Therefore, if the solution for the full fermion propagator in the problem with an external source is known and the function $\Delta(p)$ is presented, from Eq. (11) we could also obtain the dependence of the diquark field ϕ on the source. And, then, it is straightforward to calculate the generating functional by integrating the expression in Eq. (6),

$$w(j) \equiv \frac{W(j)}{\int d^4 x} = \int^{\Delta_0(j)} \phi(\Delta_0) \frac{d\Delta_0}{d\Delta_0} d\Delta_0, \quad (12)$$

where, by definition, $\Delta_0 = \Delta(p)|_{p=0}$.

The gap equation was presented in Refs. [5,7–9]. There it was also shown that the Meissner effect is of no importance for this equation. The further modification of the gap equation for the case of a nonzero external source is straightforward,

$$\Delta(p_4) \simeq j + \frac{2\alpha_s}{9\pi} \int_0^{p_4} \frac{dq_4 \Delta(q_4)}{\sqrt{q_4^2 + \Delta_0^2}} \ln \frac{\Lambda}{p_4} + \frac{2\alpha_s}{9\pi} \int_{p_4}^{\Lambda} \frac{dq_4 \Delta(q_4)}{\sqrt{q_4^2 + \Delta_0^2}} \ln \frac{\Lambda}{q_4}, \quad (13)$$

where $\Lambda = (4\pi)^{3/2} \mu / \alpha_s^{5/2}$ [8,9] and α_s is the QCD running coupling related to the scale of order μ . This equation, as is easy to check, is equivalent to the differential equation,

$$p_4 \Delta''(p_4) + \Delta'(p_4) + \frac{\nu^2}{4} \frac{\Delta(p_4)}{\sqrt{p_4^2 + \Delta_0^2}} = 0, \quad \nu = \sqrt{\frac{8\alpha_s}{9\pi}}, \quad (14)$$

along with the infrared (IR) and ultraviolet (UV) boundary conditions, $p_4 \Delta'(p_4)|_{p_4=0} = 0$ and $\Delta(\Lambda) = j$, respectively. Notice that the dependence of the solution on the source appears only through the UV boundary condition. The value of the source itself could be interpreted as the bare Majorana mass. As in Ref. [7], we solve the differential equation analytically in two regions $p_4 \ll \Delta_0$ and $p_4 \gg \Delta_0$ and, then, match the solutions at $p_4 = \Delta_0$.

In the region $p_4 \ll \Delta_0$, the solution that satisfies the IR boundary condition reads

$$\Delta(p_4) = \Delta_0 J_0 \left(\nu \sqrt{\frac{p_4}{\Delta_0}} \right), \quad (15)$$

where $J_n(x)$ is the Bessel function. In the other region, $p_4 \gg \Delta_0$, the solution, consistent with the UV boundary condition, is

$$\Delta(p_4) = B \sin \left(\frac{\nu}{2} \ln \frac{\Lambda}{p_4} \right) + j \cos \left(\frac{\nu}{2} \ln \frac{\Lambda}{p_4} \right). \quad (16)$$

Now, by matching the solutions and their derivatives at the point $p_4 = \Delta_0$, we get two relations,

$$j = \Delta_0 J_0(\nu) \cos \left(\frac{\nu}{2} \ln \frac{\Lambda}{\Delta_0} \right) - \Delta_0 J_1(\nu) \sin \left(\frac{\nu}{2} \ln \frac{\Lambda}{\Delta_0} \right), \quad (17)$$

$$B = \Delta_0 J_0(\nu) \sin \left(\frac{\nu}{2} \ln \frac{\Lambda}{\Delta_0} \right) + \Delta_0 J_1(\nu) \cos \left(\frac{\nu}{2} \ln \frac{\Lambda}{\Delta_0} \right). \quad (18)$$

The first of them relates the value of the gap in the fermion spectrum and the strength of the external source, while the other defines the integration constant B in Eq. (16). Now, we can proceed with the calculation of the generating functional. By using the relation (11), we calculate the vacuum expectation value of the diquark field,

$$\phi \simeq \frac{4\mu^2}{\pi^2} \int_0^{\Lambda} \frac{dp_4 \Delta(p_4)}{\sqrt{p_4^2 + \Delta_0^2}} = -\frac{16\mu^2}{\nu^2 \pi^2} \Lambda \Delta'(\Lambda) = \frac{8\mu^2}{\nu \pi^2} B, \quad (19)$$

with B given in Eq. (18). Then, from Eq. (12) we obtain the generating functional,

$$w = \frac{\mu^2}{\nu\pi^2} [4Bj + \nu(B^2 + j^2)], \quad (20)$$

where B should be considered as a function of j , defined by Eqs. (17) and (18). After performing the Legendre transform, Eq. (20) leads to the effective potential $V(\phi)$ in the following parametric representation:

$$V(\Delta_0) = \frac{\mu^2 \Delta_0^2}{\nu\pi^2} \left[2(J_0^2(\nu) - J_1^2(\nu)) \sin\left(\nu \ln \frac{\Lambda}{\Delta_0}\right) + 4J_0(\nu)J_1(\nu) \cos\left(\nu \ln \frac{\Lambda}{\Delta_0}\right) - \nu(J_0^2(\nu) + J_1^2(\nu)) \right], \quad (21a)$$

$$\phi(\Delta_0) = \frac{8\mu^2 \Delta_0}{\nu\pi^2} \left[J_0(\nu) \sin\left(\frac{\nu}{2} \ln \frac{\Lambda}{\Delta_0}\right) + J_1(\nu) \cos\left(\frac{\nu}{2} \ln \frac{\Lambda}{\Delta_0}\right) \right]. \quad (21b)$$

Let us study the properties of this effective potential. In order to determine the vacuum expectation value of the diquark condensate, represented by the composite field ϕ , we need to know the extrema of the potential in Eq. (21). Thus, we come to the equation $dV/d\phi = j(\Delta_0) = 0$. By solving it, we obtain an infinite set of solutions for Δ_0 ,

$$\Delta_0^{(n)} = \Lambda \exp \left[-\frac{2}{\nu} \arctan \left(\frac{J_0(\nu)}{J_1(\nu)} \right) - \frac{2\pi n}{\nu} \right] \simeq \Lambda e \exp \left[-\frac{3\pi^{3/2}(1+2n)}{23^{3/2}\sqrt{\alpha_s}} \right], \quad n = 0, 1, 2, \dots \quad (22)$$

which correspond to the following vacuum expectation values of the diquark field:

$$\phi^{(n)} = (-1)^n \frac{8\mu^2 \Delta_0^{(n)}}{\nu\pi^2} \sqrt{J_0^2(\nu) + J_1^2(\nu)}. \quad (23)$$

Since $d^2V/d\phi^2|_{\phi^{(n)}} = (\nu\pi/4\mu)^2$, we conclude that all the extrema are, in fact, minima.

It is natural to expect that the potential should also have maxima between those minima. The situation is however more subtle: while, as a function of the parameter Δ_0 , the potential does have maxima, it does not have them as a function of ϕ . Let us describe this in more detail. The first derivative of V with respect to Δ_0 is zero at the following maximum points:

$$\Delta_{0max}^{(n)} = \Lambda \exp \left[-\frac{2}{\nu} \arctan \left(\frac{J_0(\nu)}{J_1(\nu)} \right) + \frac{2}{\nu} \arctan \left(\frac{2}{\nu} \right) - \frac{2\pi n}{\nu} \right] \simeq \Lambda \exp \left(-\frac{2\pi n}{\nu} \right), \quad n = 1, 2, \dots \quad (24)$$

However, as is easy to check, the derivative of the potential with respect to the field ϕ at the corresponding ϕ -points, defined from Eq. (21b), is nonzero: it is because the derivative of ϕ with respect to Δ_0 equals zero there. As one can see from Fig. 1, this property is intimately connected with the fact that the potential $V(\phi)$ is a multibranched (multivalued) function of ϕ , and these “maximum” points are sharp turning points at which different branches of the effective potential merge.

As evident from Fig. 1, the physical branch, at which the potential takes the minimal value for a given value of ϕ , is the first branch, at which the global minimum $\phi = \phi^{(0)}$ lies¹. It is interesting that the potential is convex at this branch (we recall that the property of the convexity of an effective potential follows from general principles of quantum field theory [23]). Moreover, as is seen from Fig. 1, the potential has a fractal structure: after enlargement, the higher (“small”) branches resemble the first (“large”) branch. The whole n -th branch shrinks into the limiting point $\phi = 0$ as n goes to infinity.

As we will show below, this multivaluedness of the potential is intimately connected with the long-range nature of the interaction in the model and implies the existence of many different resonances with the same quantum numbers as the NG bosons.

The global minimum appears at $\phi^{(0)}$. In the vicinity of this minimum the approximate form of the effective potential is given by

¹Another example of a multibranched potential is connected with the θ -term in QCD: the QCD effective potential is a multibranched function of the parameter θ [22]. The physical branch is again defined as that with the minimal value of the potential for a given value of θ .

$$V(\phi) \simeq - \left(\frac{\nu\pi}{8\mu} \right)^2 \phi^2 \left[1 - \ln \left(\frac{\phi}{\phi(0)} \right)^2 \right], \quad (25)$$

i.e., in the vicinity of the minimum, it has the form of the Coleman–Weinberg potential [20]. The region of validity of this approximation is given by inequality $\nu \ln(\phi/\phi(0)) \ll 1$, and, therefore, Eq. (25) is a very good approximation for the potential of the composite field ϕ when the coupling is weak or when the value of the field is close to the minimum.

Now, let us discuss how the infinite number of minima in the effective potential (21) determine the form of the spectrum of the resonances in the channel with the quantum numbers of NG bosons. It is well known (see, for example, Ref. [24]) that, because of the Ward identities for chiral currents, the SD equation for the dynamical fermion gap coincides with the Bethe–Salpeter (BS) equation for corresponding (gapless) NG bosons, which are quark–quark bound states in the present model. The infinite number of solutions $\Delta_0^{(n)}$ (22) for the gap implies that there are massless states (which would become the NG bosons) in each of the vacua corresponding to different values of n . The genuine, stable, vacuum is that with $n = 0$. What is the fate of the quark–quark bound states which would be the NG bosons in the false vacua, with $n = 1, 2, \dots$? We will argue below that they become massive, unstable, particles there.

In the chiral limit, there are two free parameters in cold dense QCD: Λ_{QCD} and the chemical potential μ , or, equivalently, the coupling constant $\alpha_s(\mu)$ and μ . Let us consider the NG composites in a false vacuum, with $n=n^{(0)} \geq 1$. In that vacuum, they are massless bound states of fermions with the Majorana mass (gap) being equal to $\Delta_0^{(n_0)}$. The transition to the genuine vacuum, with $n = 0$, corresponds to increasing the fermion gap, $\Delta_0^{(n_0)} \rightarrow \Delta_0^{(0)}$, *without* changing the dynamics: the coupling constant $\alpha_s(\mu)$ and the chemical potential μ remain of course the same. As a result of the increase of the mass of their constituents, the square of the mass of these bound states will also increase. Therefore they become massive (apparently, unstable) composites in the genuine vacuum².

Thus we conclude that the global minimum $\Delta_0^{(0)}$ of the effective potential indeed defines the dynamical gap (Majorana mass) of fermions, and all other minima $\Delta_0^{(n)}$, $n = 1, 2, \dots$, manifest the existence of massive radial excitations of NG bosons. Notice that, because of the Higgs effect, the NG bosons are “eaten” by the five gluons, corresponding to the $SU(3)_c \rightarrow SU(2)_c$ breakdown. All the massive excitations, though, will not be affected by the Higgs mechanism.

In order to determine the spectrum of these massive excitations, one needs to study the BS equations for massive bifermion bound states in dense QCD. This problem is beyond the scope of this letter. However, it is not difficult to estimate their masses: since the fermion gap $\Delta_0^{(0)}$ is essentially the only relevant dimensional parameter in the pairing dynamics, the masses of these resonances should be of the order of the fermion gap. The resonances are unstable, although, they might be rather narrow at high density because the coupling constant is weak. The presence of such resonances would be a very clear signature of long–range forces in dense QCD.

Now we come to the description of the universality class of the dynamics in cold dense QCD. The scaling law for the order parameter is described by expression (1) with $X = \Delta_0^{(0)}$, $\Lambda_{eff} \sim \mu$, and $z = \alpha_s$. The essential singularity at $\alpha_s = 0$ is provided by long–range forces. Let us discuss the character of these forces in more detail.

The gap equation (13) in the absence of the external source can be rewritten in a different form (see Ref. [7]),

$$\Delta(p_4) \simeq \frac{4\alpha_s}{9} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \int_{-\Lambda}^{\Lambda} \frac{dq_4}{2\pi} \frac{\Delta(q_4)}{q^2 + q_4^2 + \Delta_0^2} \ln \frac{\Lambda}{|q_4 - p_4|}, \quad (26)$$

where the new integration parameter q is the spatial momentum shifted by the chemical potential, $q = |\vec{q}| - \mu$. Then, it is easy to show that Eq. (26) is equivalent to the following Schrödinger equation:

$$\left(-\frac{d^2}{d\tau^2} - \frac{d^2}{dx^2} + \Delta_0^2 + U(\tau, x) \right) \Psi(\tau, x) = 0, \quad (27)$$

where

²The quenched strong-coupling QED yields an example of a simpler model with an effective potential having the form similar to that in Eq. (21) [13,14]. The study of the BS equations in that model shows that there is indeed an infinite number of resonances in the channel with the quantum numbers of the NG bosons [24]. Their masses are nearly equal and are of the order of the fermion dynamical mass.

$$\Psi(\tau, x) = \int \frac{dp}{2\pi} \int \frac{dp_4}{2\pi} \frac{\Delta(p_4)}{p^2 + p_4^2 + \Delta_0^2} e^{ip_4\tau - ipx}, \quad (28)$$

$$U(\tau, x) = -\frac{2\alpha_s}{9\pi} \delta(x) \int_{-\Lambda}^{\Lambda} dp_4 \ln \frac{\Lambda}{|p_4|} e^{ip_4\tau} = -\frac{2\alpha_s}{9\pi|\tau|} [\pi + 2 \operatorname{si}(|\Lambda\tau|)] \delta(x). \quad (29)$$

Here $\operatorname{si}(z) = -\int_z^\infty dt \sin(t)/t$ is the sine integral function. So, we see that the problem reduces to the Schrödinger equation (27) with a non-isotropic interaction potential presented in Eq. (29). This interaction is short range in the spatial direction, x , and long-range in the (imaginary) time direction, τ [notice that $\operatorname{si}(z) \simeq -\cos(z)/z$ as $z \rightarrow +\infty$]. It is the latter long-range portion of the interaction that is responsible for the particular scaling law of the order parameter as in Eq. (1).

In some respects, the dynamics in cold dense QCD is similar to the dynamics in quenched QED₄ in a constant magnetic field [18,19]. Indeed, in both these models the dimensional reduction $3+1 \rightarrow 1+1$ in the dynamics of fermion pairing takes place. This feature and long-range interactions lead to the same scaling law (1) for the order parameter, which is qualitatively different from that of the BCS type. The form of the effective potentials in these two models is also similar (compare Eq. (21) with the expression for the potential in Ref. [16]). At the same time, the universality class of the system at hand is somewhat different from that in QED₄ in a magnetic field: in that model, the dynamics is provided by relativistic Coulomb-like forces. As is clear, the difference appears due to the explicit breakdown of Lorentz boost transformations in dense QCD by a nonzero chemical potential [notice that there is the (1+1)-dimensional Lorentz symmetry in QED₄ in a magnetic field].

In conclusion, in this letter, we have taken the first step in deriving the effective action in color superconductivity of the dense quark matter directly from QCD. In particular, we have derived the 1PI effective potential for the order parameter responsible for color symmetry breaking. In this derivation, we used the common assumption that the baryon density is high enough, so that the fermion pairing in the color antitriplet channel dominates over that in the chiral one.

The crucial feature in the dynamics of cold dense QCD is the long-range interactions mediated by the unscreened magnetic gluon modes [4,5]. Because of these long-range interactions, we argue that the system belongs to a universality class that is close to (but not quite the same as) that of quenched QED₄ in an external magnetic field [18,19].

We also argue that the spectrum of the diquark resonances with the same quantum numbers as those of NG bosons consists of a very large (infinite in our approximation) number of states with masses of order of the fermion gap. Even though these resonances are unstable, they might be relatively narrow at sufficiently high density of quark matter. We believe that the presence of such resonances would be a clear signature of the unscreened long-range forces in dense QCD. It would be worth studying in detail the properties of these resonances under the conditions produced in heavy ion collisions: if the color superconducting phase is ever going to be produced in heavy ion collisions, the detection of these diquark resonances might be a crucial piece of information for determining the nature of the phase. In further studies, it would be interesting to clarify the properties of resonances in other channels as well as to derive the effective potential in the case of intermediate densities, when the chiral and the diquark condensates compete [3,4].

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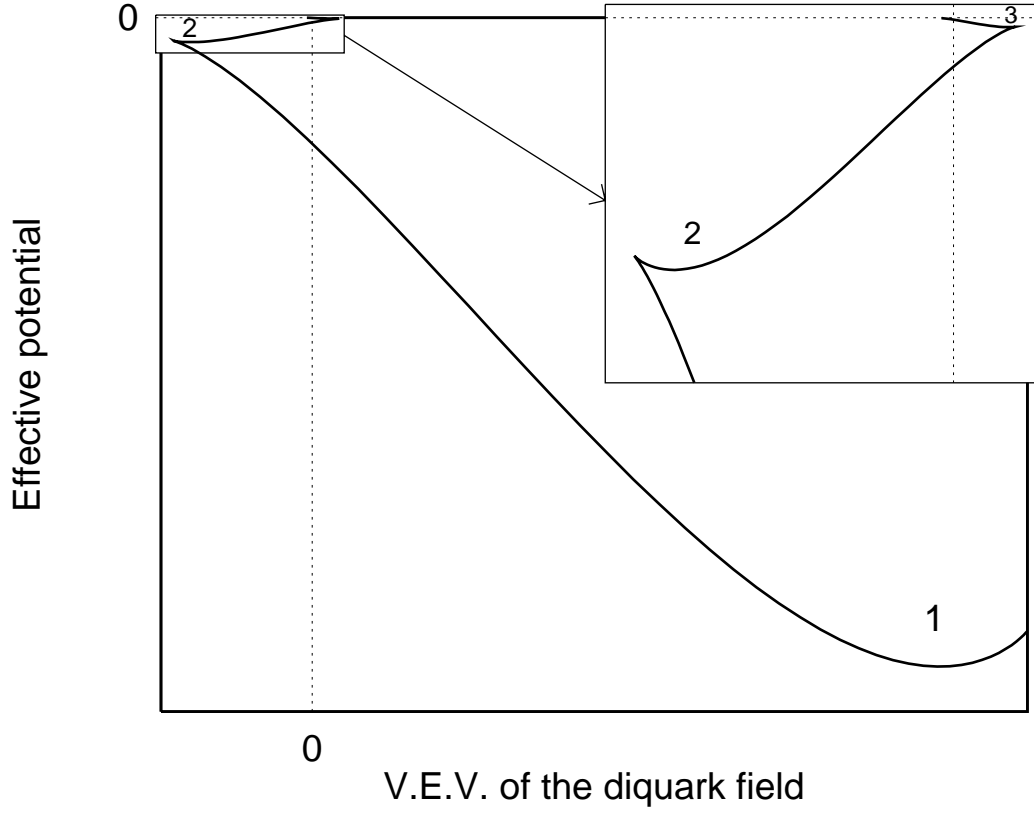


FIG. 1. An illustration of the multibranched structure of the effective potential as a function of ϕ . The enlargement of the higher (second and third) branches is also shown